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## **ORBIT-3.0 — A COMPUTER CODE FOR SIMULATION AND CORRECTION OF THE CLOSED ORBIT AND FIRST TURN IN SYNCHROTRONS**

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A new computer program ORBIT-3.0 for simulation and correction of the closed orbit and first turn in synchrotrons is described. The program works under WINDOWS 95/98/NT and is a full object oriented application. It has an interactive interface, enhanced graphical capabilities and on-line printing. The original algorithms DINAM — for closed orbit correction and FTURN — for first turn steering are described as well.

The investigation has been performed at the Institute for Nuclear Research and Nuclear Energy, Bulgaria.

## **ORBIT-3.0 — компьютерная программа для моделирования и коррекции замкнутой орбиты и первого оборота в синхротронах**

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Описана компьютерная программа ORBIT-3.0 для моделирования и коррекции замкнутой орбиты и первого оборота в синхротронах. Программа работает под WINDOWS 95/98/NT и полным набором ориентированных приложений. Она представляет собой интерактивный интерфейс, дополненный графическими возможностями с он-лайн распечаткой. Также описаны первоначальные алгоритмы DINAM — для замкнутой орбиты и FTURN — для управления первым оборотом.

Работа выполнена в Институте ядерных исследований и ядерной энергии, Болгария.

### **1. INTRODUCTION**

One of the major tasks of the optimum operation of a cyclic charged particle accelerator of synchrotron type is the control and the correction of the equilibrium closed orbit. Due to the errors in the dipole magnetic fields, random transversal displacements of the quadrupoles from the reference orbit, random tilts of the dipole magnets around the longitudinal axis, stray magnetic fields and ground movements the real closed orbit is distorted. The maximum

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deviations of the distorted orbit from the reference orbit may reach some ten millimetres. Such large deviations do not allow one to use effectively the accelerator aperture and destroy the optimum work of the injection and extraction systems. For these reasons synchrotrons have special magnetic systems for correcting the closed orbit.

Special orbit correction algorithms have been developed in order to set the proper strengths of the orbit correctors [1]. Each of these algorithms has its specific advantages and disadvantages.

One of the major tasks at the design time of any synchrotron is to assess the most probable mean orbit deviation and the orbit amplitude as well. Several approximate analytical estimations have been found [2], but the computer simulations still remain a safer approach. Knowing these estimations helps us to design an adequate orbit correcting system.

A special problem is the first turn steering.

During the assembling and initial tuning of the accelerator much bigger errors than the usual random field and alignment errors may occur. Sometimes they are caused by unpredictable mistakes and there are several such cases in the accelerator practice. In the presence of big linear errors the centre of charge trajectory does not follow any more the orbit and can have big deviations from it. Even more, the beam can hit somewhere the vacuum chamber not being able to make a complete turn around the machine. Another case is when the first turn doesn't close onto the second. Along with the launch errors during injection this may cause harmful coherent oscillations of the beam.

Many of the widespread computer codes created for general accelerator design — MAD [3], DIMAD [4], PETROC [5] and others — possess features for calculating the closed orbit under the influence of random errors and incorporate one of the orbit correction methods. However these programs have no build-in capability for making statistics both of orbit and of its correction over an ensemble of some hundred or some thousand random machines. Such feature is very helpful during the accelerator design.

It is also advisable to compare different correction algorithms by means of computer simulations before to make your choice in favour of one of them.

In addition the problem of first turn threading and closing lies beyond the scope of interest of the above-mentioned general purpose programs. Specialized computer codes have been developed for first turn steering [6,7].

Last but not least is the old fashion interface of all these programs. Designed as a rule in the 70s for the monstrous old computers and gradually improved in the time these general accelerator design programs are not interactive and have limited graphical capabilities (usually a sort of off-line graphics). They are far from the now standard Windows-type interface as well.

In 1991, I developed the computer code ORBIT-2.0, especially intended for closed orbit simulation and correction. The computer code was created for the needs of the COLer SYnchrotron COSY-Julich [7]. ORBIT-2.0 was written in C and runs on PCs under MS DOS. This is a menu driven program with enhanced graphical capabilities. A UNIX version with off-line graphics and printing was developed as well. Now a full MS WINDOWS 95/98/NT computer code ORBIT-3.0 for simulation and correction of both closed orbit and first turn is available.

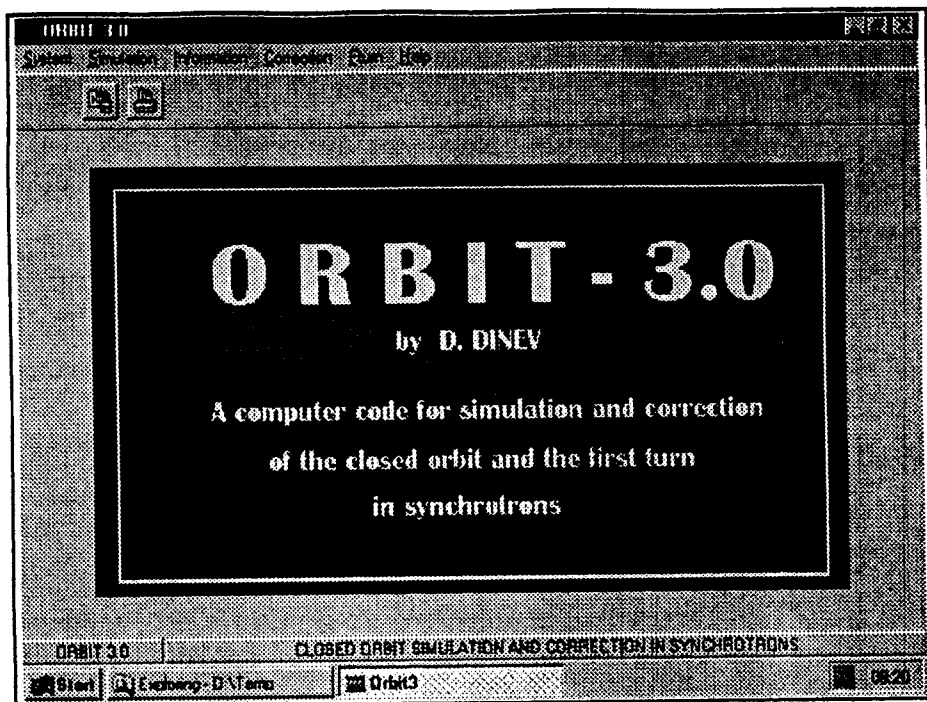


Fig. 1. Computer code ORBIT-3.0

## 2. COMPUTER CODE ORBIT-3.0

**2.1. General Description.** The computer code ORBIT-3.0 is a full MS WINDOWS 95/98/NT application. It is written in C++ and uses all the advantages of the object oriented programming. As a windows application ORBIT-3.0 has access to the big collection of WINDOWS resources and especially on-line printing, of both text and graphics on all the installed printers. The program has an interactive interface and enhanced graphical capabilities.

**2.2. Main Options.** The ORBIT-3.0 options are grouped in four groups: closed orbit simulation and statistics; processing of real orbit measurement data in order to produce some kind of additional information; correction of the measured or simulated orbits; first turn steering (Fig.1).

**2.3. Orbit Simulation.** As is well known the equation of transverse particle motion under linear perturbations can be put in the form of a forced oscillation equation by proper choice of the variables [1]. The  $2\pi$ -periodic solution, i.e., the closed orbit can be represented by the formula:

$$\eta = \frac{Q}{2 \sin \pi Q} \int_{\phi}^{\phi+2\pi} f(t) \cos Q(\phi + \pi - t) dt. \quad (1)$$

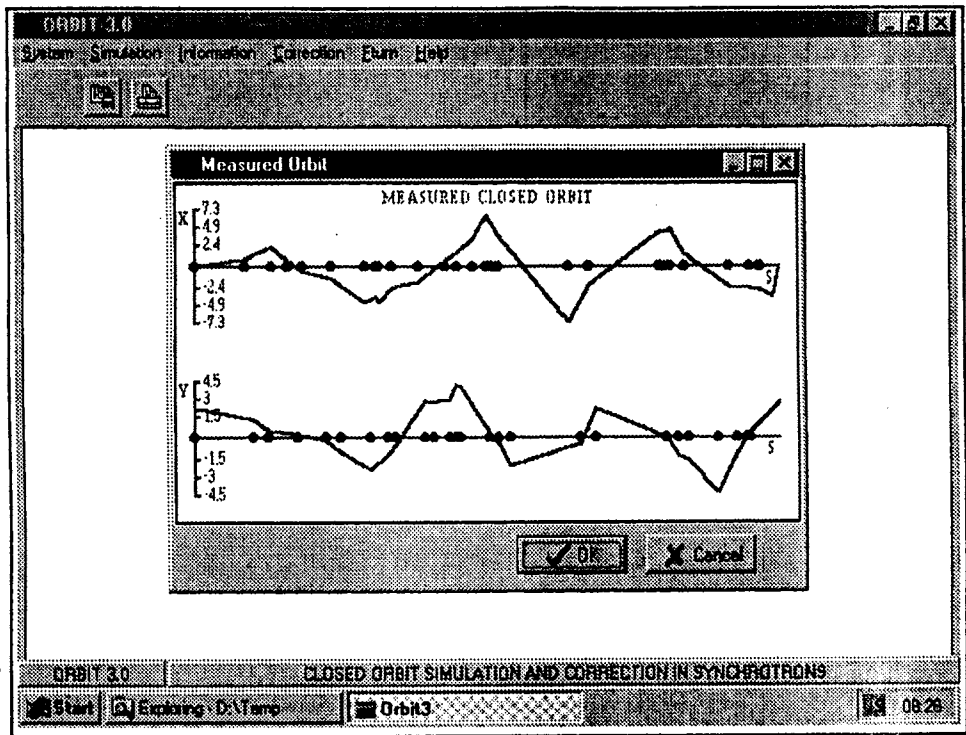


Fig. 2. Graphical representation of the closed orbit

In (1)  $f(t)$  is a function which describes the linear perturbations,  $Q$  is the tune and:

$$\eta = \frac{x}{\sqrt{\beta(s)}}, \quad \phi = \int_0^s \frac{ds}{Q\beta(s)}, \quad (2)$$

$\beta(s)$  being the Twiss's amplitude function,  $x$  — the transverse and  $s$  — the longitudinal coordinates. It is very important to obtain a realistic predictions about the closed orbit behaviour in the future machine during the design period. The aperture of the vacuum chamber and the maximum strengths of the orbit correctors are determined on the base of the rms and maximum orbit deviations from the reference orbit. Knowing from the magnet measurements the dipole field errors  $\Delta B$  and from the geodesy the expected transverse misalignments  $\Delta x$  and  $\Delta z$  and tilts  $\Delta\theta$  of the elements one can calculate the statistical characteristics of the future orbit. In ORBIT-3.0 this is done either analytically applying the formulae obtained in [2] or by means of computer simulations. In [2] the variance of the maximum orbit amplitude is calculated taking into account only the major four orbit harmonics. ORBIT-3.0 calculates the value of the maximum orbit amplitude that will not be exceeded by 98% probability, the mean and the maximum rms deviations, and the rms orbit amplitude.

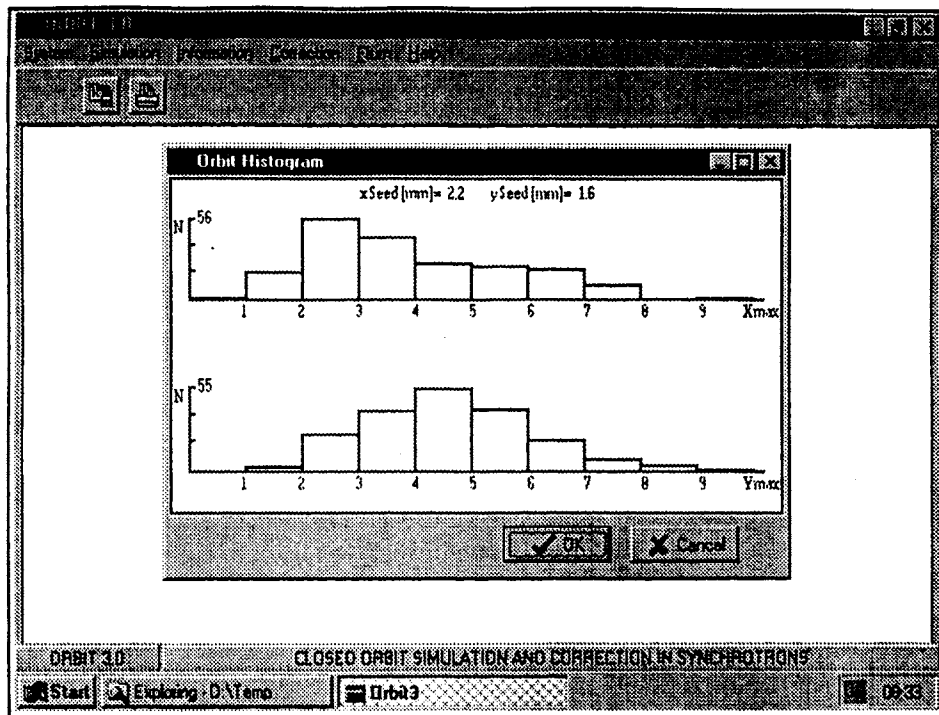


Fig. 3. Histograms of the simulated closed orbit

Another approach used for making orbit statistics is the simulation. An ensemble of identical machines with errors generated by a random number generator with Gaussian distribution is investigated. Closed orbits in each machine are represented either by tables of orbit deviations in beam position monitors (BPM) and structure elements or graphically (Fig.2). The results of orbit statistics are shown as histograms of maximum orbit amplitude distribution over the ensemble (Fig.3).

The Fourier spectrum of the simulated orbit is calculated through the well-known relation between harmonics of the perturbations  $f_k$ ,  $g_k$  and orbit  $u_k$ ,  $v_k$ :

$$u_0 = v_0, \quad u_k = \left( \frac{Q^2}{Q^2 - k^2} \right) f_k, \quad v_k = \left( \frac{Q^2}{Q^2 - k^2} \right) g_k. \quad (3)$$

**2.4. Orbit Information.** This group of options embraces features for processing the data of real orbit measurements in order to produce some sort of additional information.

First of all it is possible for the picture of the measured orbit along the machine circumference to be shown on the screen and printed in a selected printer.

A task that one frequently faces in the practice is to calculate the orbit deviations between BPMs. ORBIT-3.0 offers two possibilities. In local interpolation mode the orbit deviations in some number of points along the circumference are calculated applying several interpolation

methods. In general interpolation mode, the orbit in all the structure elements of the machine is restored on the base of BPMs measurements.

Finally the Fourier spectrum of the measured orbit could be calculated trough FFT and shown on the screen.

**2.5. Orbit Correction.** ORBIT-3.0 includes six algorithms for orbit correction: harmonic correction, beam-bump, least-squares (LSQ), singular value decomposition (SVD), Baconier's method and DINAM. More information about these and other correction methods can be found in the review paper [1].

Harmonic correction is realised with the opportunity to set individual weights for each harmonic.

Beam-bump correction is coded for the case of arbitrary deployed correctors and monitors.

LSQ correction uses either LU matrix decomposition or Gauss-Jordan elimination methods for response matrix inversion.

SVD correction makes use of the singular value decomposition of the response matrix  $A$ :

$$A = U W V^T, \quad (4)$$

where  $U$  and  $V$  are unitary matrix and  $W$  is the diagonal matrix of the singular values  $w_i$  [9].

Obtaining the above decomposition of the response matrix, the vector of the corrections  $\vec{\delta}$  is calculated through:

$$\vec{\delta} = -V[\text{diag}(1/w_i)]U^T\vec{\eta}, \quad (5)$$

$\vec{\eta}$  being the vector of orbit deviations in BPMs. It should be emphasised that in case of some  $w_i$ ,  $s$  equal to zero or very small (singular or ilconditioned response matrix), the corresponding diagonal elements ( $1/w_i$ ) in (5) must be replaced by zero ( $\infty \rightarrow 0!$ ).

In Baconier's correction method the goal function is:

$$q = \gamma \sum_{n=1}^N \eta_n^2 + (1 - \gamma) \sum_{j=1}^K \delta_j^2 \Rightarrow Min, \quad (6)$$

where  $\eta_i$  are the orbit deviations in BPMs,  $\delta_j$  are the corrector strengths and  $\gamma$  is a parameter,  $0 < \gamma < 1$ .

As (6) reads, both the orbit deviations in BPMs and the strengths of the correctors are restricted thus reducing the influence of the remaining uncorrected higher harmonics of the correction.

DINAM is an original algorithm for orbit correction developed by the author and it will be described wider in the next chapter.

**2.6. First Turn Steering.** As we have mentioned in the introduction the first turn correction is a separate problem which frequently arises during the machine commitioning.

First of all we should thread the beam making it to perform a full turn around the accelerator. Having corrected the centre of charge trajectory so that the beam goes entire turn around the ring we should close this trajectory, i.e., make the second turn to coincide with the first one.

ORBIT-3.0 includes options for both first turn threading and first turn closing. The algorithm applied for this in ORBIT-3.0 will be described in the last chapter of this paper.

### 3. THE DINAM CORRECTION ALGORITHM

While the other orbit correction algorithms, described in detail in [1], strive to compensate the orbit deviations only in the points where BPMs are situated, the DINAM algorithm corrects the orbit over the whole accelerator ring. For that reason the criterion of correction quality is defined as the functional  $q = (1/2\pi) \int_0^{2\pi} \eta^2(\phi) d\phi$  [10], where  $\eta$  is the orbit and  $\phi$  is machine azimuth.

As the orbit  $\eta(\phi)$  is a random function of the azimuth we should improve this criterion by taking the mathematical expectation of the functional:

$$q = M\left[\left(\frac{1}{2\pi} \int_0^{2\pi} \eta^2(\phi) d\phi\right)\right] \quad (7)$$

With this choice of the correction quality we exclude the cases when the corrected orbit has approximately zero deviations in BPMs and relatively big deviations in between.

It is shown that the quality criterion can be put in the following form [10]:

$$q = \frac{1}{N} \sum_{i=1}^N \eta_i^2 + \frac{1}{2} \sum_{p=1}^N \sum_{q=1}^K A_{pq} \eta_p \delta_q + \frac{1}{2} \sum_{q=1}^K K \sum_{r=1}^K B_{qr} \delta_q \delta_r, \quad (8)$$

where  $\eta$  is the orbit,  $\eta$  is the corrector strength,  $N$  is number of BPMs and  $K$  is the number of orbit correctors. The coefficients  $A_{pq}$  and  $B_{qr}$  in (8) describe the optical parameters of the correction system and depend on  $\phi_p$ , the azimuths of the BPMs and  $\phi_q$ ,  $\phi_r$ , the azimuths of the correctors:

$$A_{pq} = \frac{2}{N} \left( c_0 + 2 \sum_{k=1}^{n/2-1} c_k \cos k(\phi_p - \phi_q) + c_{N/2} \cos \frac{N}{2} \phi_p \cos \frac{N}{2} \phi_q \right), \quad (9)$$

$$B_{qr} = \frac{c_0^2}{2} + \sum_{k=1}^{\infty} c_k^2 \cos k(\phi_q - \phi_r), \quad (10)$$

where:

$$c_k = \frac{2 \sin \pi Q}{\pi^2(Q^2 - k^2)}. \quad (11)$$

The strengths of the orbit correctors are determined by the condition of minimum of  $q$  to occur. This leads to the following matrix equation for the correction vector  $\vec{\delta}$ :

$$\vec{\delta} = R\eta. \quad (12)$$

The matrix

$$R = -\frac{1}{2} A^{-1} B^T, \quad (13)$$

where  $A = \{A_{pq}\}$  and  $B = \{B_{pq}\}$ .

The matrix depends only on the azimuths of the BPMs and of the correctors and for the given accelerator can be calculated prior to the correction.

Figure 4 shows an example of DINAM orbit correction.

The DINAM correction algorithm is relatively fast. It works reliably. It was used in the COSY-Julich and JINR-Nuclotron synchrotrons.

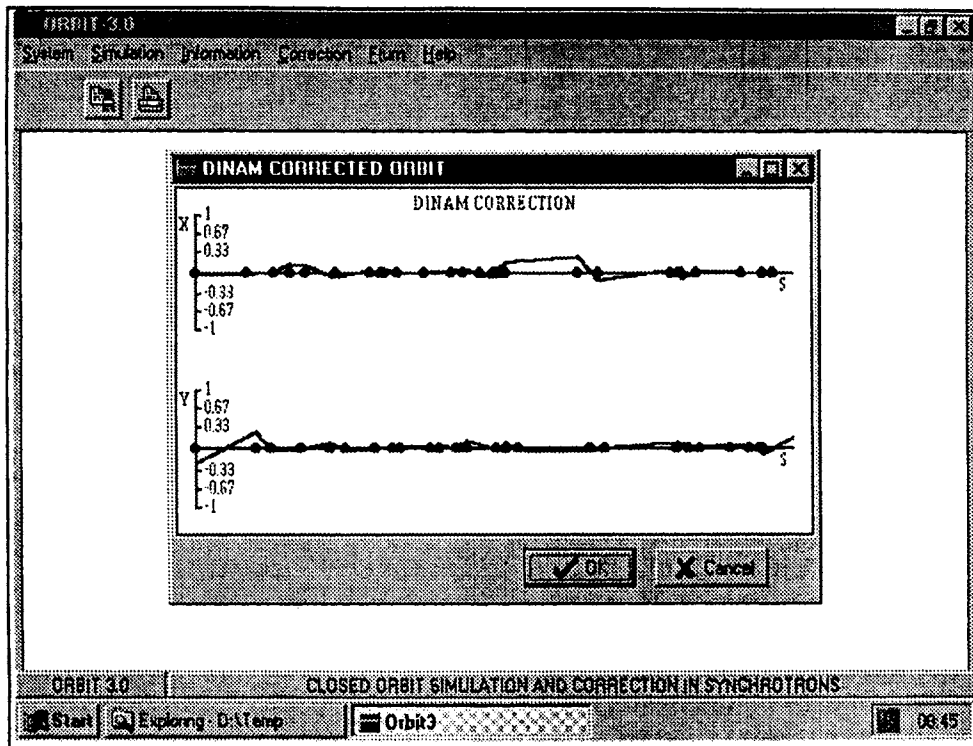


Fig. 4. DINAM closed orbit correction

#### 4. FIRST TURN STEERING

In order to thread the beam entire turn around the accelerator ring and make the centre of charge trajectory to follow the injection orbit as close as possible an original first turn steering algorithm has been developed for ORBIT-3.0.

The algorithm uses the least squares approach but allows for constraints on the corrector kicks to be set.

The first turn correction is closely related to the beam line steering. In fact before closing the first turn onto the second the magnetic structure of a circular accelerator can be look at as a beam line. The BPMs «see» only those correctors which are deployed in front of them.

Applying the superposition principle one can write for the orbit deviation produced by the correctors in the  $i$ -th BPM:

$$\eta_i = \sum_{j=1}^k a_{ij} \epsilon_j^*, \quad (14)$$

where:

$$a_{ij} = \sin Q(\phi_i^{BPM} - \phi_j^C) e^{i(\phi_i^{BPM} - \phi_j^C)}, \quad (15)$$



$k$  being the number of used correctors and  $e$  — the Heaviside step function

$$\epsilon^* = \sqrt{\beta^C} \epsilon, \quad (16)$$

$\epsilon$  being the corrector kick.

Our algorithm follows the least squares approach.

In practice very often the available corrector strengths are not big enough for the exact correction to be realized. For that reason we have added parameter constraints to our algorithm. We used the penalty function method for that.

The goal function chosen is:

$$\Psi(\epsilon_j^*) = \sum_{i=1}^n \left[ \eta_i^{BPM} - \eta_i^{des} + \sum_{j=1}^k a_{ij} \epsilon_j^* \right]^2 \Rightarrow \min. \quad (17)$$

In (17)  $\eta_i^{des}$  denotes the desired centre of charge trajectory;  $n$  and  $k$  are the current numbers of used monitors and correctors. In the case when we have lost the beam before a full turn to be carried out the current number of used monitors  $n$  is less than the total number of monitors and the current number of switched on correctors  $k$  is less than the total number of available correctors  $K$ .

We should look for the minimum of (17) to occur under the following constrains on the corrector kicks:

$$|\epsilon_j| < \Delta, \quad j = 1, \dots, k. \quad (18)$$

Thus we face a constrained optimization problem [11].

In order to solve this problem we apply the penalty function method. According to this method we reduce the constrained optimization problem to a series of unconstrained problems by adding to the goal function (17) the so-called penalty function  $\alpha(\epsilon^*)$ :

$$\Psi(\epsilon_j^*, \mu_k) = \Psi(\epsilon_j^*) + \mu_k \alpha(\epsilon_j^*) \Rightarrow \min, \quad (19)$$

where  $\mu_k > 0$ ,  $k = 1, 2, 3, \dots$  are parameters and:

$$\lim_{k \rightarrow \infty} \mu_k = \infty. \quad (20)$$

The penalty function  $\alpha(\epsilon^*)$  must «punish» the function  $\Psi(\epsilon^*)$  if the constraints (18) are broken and in ORBIT-3.0 we have chosen that:

$$\alpha(\epsilon_j^*) = \sum_{j=1}^k \max(0, \epsilon_j^{*2} - \Delta_j^2), \quad (21)$$

$$\Delta_j = \Delta \sqrt{\beta_j^C}. \quad (22)$$

Let  $\epsilon_{jk}^*$ ,  $k = 1, 2, 3, \dots$  be the consecutive points of minimum of  $\Psi(\epsilon_j^*, \mu_k)$ . It can be proved [11] that:

$$\lim_{k \rightarrow \infty} \epsilon_{jk}^* = \epsilon_j^*_{opt} \quad (23)$$

and:

$$\lim_{k \rightarrow \infty} [\min \Psi(\epsilon_j^*, \mu_k)] = \Psi(\epsilon_{j \text{ opt}}^*), \quad (24)$$

where  $\epsilon_{j \text{ opt}}^*$  be the minimum of the goal function (17) under the constraints (18). After threading the beam entire turn around the accelerator we should ensure that the beam closes itself, i.e., that the second (and the following turns) coincide with the first turn. To perform the closing we need information about the first and second turn deviations in the first two BPMs. This information can be provided by these monitors if they work in a single turn mode. The beam closing in ORBIT-3.0 is achieved by appropriate tuning of the last two correctors in the ring.

### References

1. Dinev D — Closed Orbit Correction in Synchrotrons. Physics of Particles and Nuclei, 1997, v.28, No.4, pp.1013—1060.
2. Gluckstern R. — Particle Accelerators, 1978, v.8, p.203.
3. Iselin F.C., Grote H. — The MAD Program — User's Reference Manual. Version 8.10. CERN No. SL 90 — 13, 1993.
4. Servranckx R., Brown K., Schachinger L., Douglas D. — User's Guide to the Program DIMAD. SLAC Report No. 285 UC — 28, 1985.
5. Guignard G., Marty Y. — CERN, Preprint No. LEP — TH/83 — 50, 1983.
6. Raya R., Rusell A., Aukenbrandt C. — Nuclear Instruments and Methods, 1989, v.A242, p.15.
7. Paxson V., Peggs S., Schachinger L. — Interactive First Turn and Global Orbit Correction. First European Particle Accelerators Conference EPAC—88, Rome, 1988, p.824.
8. Maier R.. Status of COSY. — Fourth European Particle Accelerators Conference EPAC—94, London, 1994.
9. Chung Y., Decker G., Evans K. — Proceedings of IEEE Particle Accelerators Conference, Washington, 1993.
10. Dietrich J., Dinev D., Martin S., Wagner R. — Simulation and Correction of the Closed Orbit in the Cooler Synchrotron COSY — Julich. Third European Particle Accelerators Conference EPAC—92, Berlin, 1992.
11. Gill P. E., Murray W., Wainwright M. H. — Practical Optimization. Academic Press, New York, 1982.